

# VEER NARMAD SOUTH GUJARAT UNIVERSITY

## M. Sc. ( Part – I)

### Mathematics

w.e.f. 2002

### Paper 401 Real Analysis

L	P	T	
<b>Total</b>			
<b>4</b>	<b>0</b>	<b>0</b>	<b>4</b>

#### Lebesgue Measure and Lebesgue Integral:

Introduction, outer measure, measurable sets and Lebesgue measure, a non-measurable set, measurable functions, Littlewood's three principles, the Riemann integral, the Lebesgue integral of a bounded function over a set of finite measure, the integral of a non-negative function, the general Lebesgue integral, convergence in measure.

#### Differentiation and Integration:

Differentiation of monotone functions, functions of bounded variation, differentiation of an integral, absolute continuity, convex functions.

#### The Classical Banach Spaces:

The  $L^p$  spaces, the Minkowski and Holder inequalities, convergence and completeness, approximation in  $L^p$ , bounded linear functions on the  $L^p$  spaces.

#### Measure and Integration:

Measure spaces, measurable functions, integration, general convergence theorems, signed measures, the Radon-Nikodym theorem, the  $L^p$  spaces.

#### Measure and Outer Measure:

Outer measure and measurability, the extension theorem, the Lebesgue-Stieltjes integral, product measure, integral operations, inner measure, extension by sets of measure zero, Carathéodory outer measure, Hausdorff measure.

#### **References:**

1. H. L. Royden, Real Analysis, Macmillan Pub. Co. Inc. 4<sup>th</sup> Edition, New York, 1993.
2. Walter Rudin, Principles of Mathematical Analysis (3<sup>rd</sup> Edition) Mc Graw Hill, Kogakusha, 1976, International Student Edition.
3. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
4. Gabriel Klambauer, Mathematical Analysis, Marcel Dekker, Inc. New York, 1975.
5. A. J. White, Real Analysis; an Introduction, Addison-Wesley Publishing Co., Inc. 1968.
6. G. de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.
7. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Berlin, Springer, 1969.
8. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd. Published, New Delhi, 1986 (Reprint 2000).

9. I. P. Natanson, Theory of Functions of a Real Variable, Vol. I, Frederick Ungar Publishing Co., 1961.
10. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An Introduction to Real Analysis, Marcel Dekker Inc. 1977.
11. J. H. Williamson, Lebesgue Integration, Holt Rinehart and Winston, Inc. New York, 1962.
12. A. Friedman, Foundations of Modern Analysis, Holt, Rinehart and Winston, Inc. New York, 1970.
13. P. R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
14. T. G. Hawkins, Lebesgue's Theory of Integration: Its Origins and Development, Chelsea, New York, 1979.
15. K. R. Parthasarathy, Introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
16. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966.
17. Serge Lang, Analysis I & II, Addison-Wesley Publishing Company, Inc. 1969.
18. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.
19. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 1966.

# VEER NARMAD SOUTH GUJARAT UNIVERSITY

## M. Sc. ( Part – I)

### Mathematics

#### Paper 402

#### Functional Analysis

L	P	T	
<b>Total</b>			
<b>4</b>	<b>0</b>	<b>0</b>	<b>4</b>

#### Normed Spaces and Banach Spaces:

Vector spaces, Brief introduction to convergence and completeness in  $\mathbb{R}^n$ . Introduction to basic results of Linear Algebra, Definition of norm and Banach space with illustrations, Properties of normed spaces, Theorem of completion on normed spaces (without proof), Finite dimensional normed spaces and subspaces, Compactness and finite dimension, Riesz lemma, Linear operators and their illustrations, Bounded continuous linear operators and their illustrations, Linear functionals and illustrations, Algebraic dual space and reflexivity, Functionals on finite dimensional spaces, Normed spaces of operators and dual space.

#### Inner Product Spaces – Hilbert Space:

Inner product space – definition and illustrations, Hilbert space, Properties of inner product spaces, Orthogonal complements and direct sums, Orthogonal sets, sequences and related series, Total orthonormal sets and sequences, Functionals on Hilbert spaces, Riesz's theorem, Sesquilinear form and Riesz representation theorem, Hilbert adjoint operator and its properties, Self adjoint, unitary and normal operators.

#### Contraction Mapping Principle:

Banach fixed-point theorem, Application of Banach fixed-point theorem to system of linear equations, differential equations and integral equations.

#### **References:**

1. Introduction to Functional Analysis with Applications by Erwin Kreyszig. John Wiley & Sons, 1978.
2. Functional Analysis by B. V. Limaye. 2<sup>nd</sup> ed. New age Int. Pvt. Ltd.
3. Introduction to Topology and Modern Analysis by G. F. Sommons. McGraw Hill Book Co.
4. Functional Analysis by Koffman and Patric.
5. Functional Analysis by A. H. Siddiqui. Prentice Hall of India.
6. Functional Analysis by Sudarshan Nanda. Wiley Estarn Pvt. Ltd.
7. Normed Linear Spaces by M. M. Day. 3<sup>rd</sup> ed. 1973. Springer-Verlag.

# VEER NARMAD SOUTH GUJARAT UNIVERSITY

## M. Sc. ( Part – I)

### Mathematics

#### Paper 403

#### Topology

L	P	T	
<b>Total</b>			
<b>4</b>	<b>0</b>	<b>0</b>	<b>4</b>

#### Metric Spaces:

The definition and some examples, open sets, closed sets, convergence, completeness, and Baire's theorem, continuous mapping, spaces of continuous functions, Euclidean and unitary spaces.

#### Topological Spaces:

The definition and some examples, elementary concepts, open bases and sub-bases, weak topologies, the function algebras of  $C(X, \mathbb{R})$ , and  $C(X, \mathbb{C})$ .

#### Compactness:

Compact spaces, product spaces, Tychonoff's theorem and locally compact spaces, compactness of metric spaces, Ascoli's theorem.

#### Separation:

Separation axioms,  $T_0$ ,  $T_1$ ,  $T_2$ ,  $T_{3\frac{1}{2}}$ ,  $T_4$ ,  $T_1$ -spaces and Hausdorff spaces, completely regular spaces and normal spaces. Urysohn's lemma and the Tietze extension theorem, the Urysohn's imbedding theorem, the Stone-Cech compactification.

#### Connectedness:

Connected spaces, the components of a space, totally disconnected spaces, locally connected spaces.

#### Approximation:

The Weierstrass approximation theorem, the Stone-Weierstrass theorem, locally compact Hausdorff spaces, the extended Stone-Weierstrass theorems.

#### **References:**

1. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Co., 1963.
2. James R. Munkres, Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
3. J. Dugundji, Topology, Allyn and Bacon, 1966 (Reprinted in India by Prentice Hall of India Pvt. Ltd.).
4. K. D. Joshi, Introduction to General Topology, Wiley Eastern td., 1983.
5. J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1961.

6. J. L. Kelley, *General Topology*, Van Nostrand, Reinhold Co., New York, 1955.
7. L. Steen and J. Seebach, *Counter examples in Topology*, Holt, Rinehart and Winston, New York, 1970.
8. W. Thron, *Topological Structures*, Holt, Rinehart and Winston, New York, 1966.
9. N. Bourbaki, *General Topology Part I* (Transl.) Addison Wesley, Reading, 1966.
10. R. Engelking, *General Topology*, Polish Scientific Publishers, Warszawa, 1977.
11. W. J. Pervin, *Foundations of General Topology*, Academic Press Inc. New York, 1964.
12. E. H. Spanier, *Algebraic Topology*, McGraw-Hill, New York, 1966.
13. S. Willard, *General Topology*, Addison-Wesley, Reading, 1970.
14. Crump W. Baker, *Introduction to Topology*, Wm C. Brown Publisher, 1991.
15. Sze-Tsen Hu, *Elements of General Topology*, Holden-Day, Inc. 1965.
16. D. Bushaw, *Elements of General Topology*, John Wiley & Sons, New York, 1963.

# VEER NARMAD SOUTH GUJARAT UNIVERSITY

## M. Sc. ( Part – I)

### Mathematics

#### Paper 404

#### Complex Analysis

L	P	T	
<b>Total</b>			
<b>4</b>	<b>0</b>	<b>0</b>	<b>4</b>

#### Complex Numbers and Functions:

Introduction, polar form, complex valued functions, limit, complex differentiability, Cauchy-Riemann equations.

#### Power Series:

Formal power series, convergent power series and relation between them, analytic functions, differentiation of power series.

#### Cauchy Theorems and Winding Numbers:

Holomorphic functions on connected sets, integrals over paths, local primitive for a Holomorphic function, the Homotopy form of Cauchy's theorem, global primitives, definition of the logarithm, the local Cauchy formula, Winding numbers, the global Cauchy theorem.

#### Applications of Cauchy Integral Formula & Calculus of Residues:

Uniform limits of analytic functions, Laurent series, isolated singularities, removable singularities, poles and essential singularities, the Cauchy residue theorem, residues of differentials, evaluation of definite integrals, fourier transforms, trigonometric integrals, Mellin transform.

#### Harmonic Functions:

Definition, examples, basic properties, the poisson formula and construction of harmonic functions.

#### Entire and Monomorphic and Elliptic Functions:

Jensen's formula and inequality, definition of entire functions, infinite products, Weierstrass products, functions of finite order, meromorphic functions, Mittag-Leffler theorem, the Liouville theorems, the Weierstrass function, the addition theorem, sigma and zeta functions, gamma functions.

#### **References:**

1. S. Lang, Complex Analysis, Addison Wesley, 1997.
2. S. Ponnuswamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.
3. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.

4. J. B. Conway, Functions of one Complex variable, Springer-Verlag, International Student Edition, Narosa Publishing House, 1980.
5. Liang-Shin Hahn & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
6. L. V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.
7. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
8. Mark J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
9. E. Hille, Analytic Function Theory (2 Vols.), Gonn & Co., 1959.
10. W. H. J. Fuchs, Topics in the Theory of Functions of One Variable, D. Van. Nostrand Co., 1967.
11. C. Caratheodory, Theory of Functions (2 Vols.), Chelsea Publishing Company, 1964.
12. M. Heins, Complex Function Theory, Academic Press, 1968.
13. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
14. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952.
15. E. C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
16. W. A. Veech, A Second Course in Complex Analysis, W. A. Benjamin, 1967.

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## M. Sc. ( Part – I)

### Mathematics

#### Paper 405

#### Differential Equations

L	P	T	
<b>Total</b>			
<b>4</b>	<b>0</b>	<b>0</b>	<b>4</b>

#### Existence, Uniqueness, and Continuation of Solutions:

Introduction, Notation and Definitions, Existence and Uniqueness of Solutions of Scalar Differential Equations, Existence Theorems for System of Differential Equations, Differential and Integral Inequalities.

#### Linear Systems:

Introduction, Properties of Linear Homogeneous Systems, Inhomogeneous Linear Systems, Behaviour of Solutions of  $n^{\text{th}}$  Order Linear Homogeneous Equations.

#### Stability of Linear and Weakly Nonlinear Systems:

Introduction, Continuous Dependence and Stability Properties of Solutions, Linear Systems.

#### Second Order Differential Equations:

Introduction, Preliminary Results, Boundedness of Solutions, Application to Some Classical Equations.

#### Mathematical Models in Population Dynamics:

Introduction, Single-Species Models, Two-Species Lotka-Volterra Models, Multi-Species Models.

#### **References:**

1. Shair Ahmad and M Rama Mohana Rao, Theory of Ordinary Differential Equations, Affiliated East-West Press Pvt. Ltd., New Delhi, 1999.
2. P. Hartman, Ordinary Differential Equations, John Wiley, 1964.
3. W. T. Reid, Ordinary Differential Equations, John Wiley, New York, 1971.
4. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw-Hill, NY, 1955.